

# LETTERS TO THE EDITOR

To the editor:

On "Transition to Annular Flow in Vertical Upward Gas-Liquid Flow" by Chen and Spedding [*AIChE J.*, 29, 525 (1983)]

A force balance between the gravity and drag forces acting on a spherical liquid drop suspended in a gas stream yields the minimum gas velocity required to suspend the drop [Taitel et al., (*AIChE J.*, 26, 345 (1980))]

$$U_G = \frac{2}{\sqrt{3}} \left[ \frac{gd(\rho_L - \rho - \rho_g)}{\rho_g C_d} \right]^{1/2} \quad (1)$$

where the drag coefficient,  $C_d$ , depends, in general, on the drop Reynolds number,  $Re_p = \rho_g d U_G / \mu_g$ , and the ratio of the drop diameter to the pipe diameter. Taitel et al (1980) used Hinze's (1955) analysis to obtain the maximum stable drop size in the form

$$d = \frac{K\sigma}{\rho_g U_G^2} \quad (2)$$

where  $K$  is the critical Weber number which takes a value between 20 and 30. Hinze's analysis is based on the premise that the maximum stable drop size is determined by the balance between the impact force of the gas that tends to shatter the drop (which is proportional to  $\rho_g U_G^2$ ) and surface tension forces that hold the drop together (which are proportional to  $d/\sigma$ ).

Combining equations, Taitel et al obtained the following relation for the annular flow transition velocity

$$U_G = \left( \frac{4K}{3C_d} \right)^{1/4} \left[ \frac{\sigma g(\rho_L - \rho_g)}{\rho_g^2} \right]^{1/4} \quad (3)$$

Taitel et al choose the largest  $K$  value and the smallest  $C_d$  value (applicable to high  $Re_p$ ), thus maximizing  $U_G$ . In addition, their final annula flow transition boundary relation uses  $U_{SG}$  in place of  $U_G$  which further tends to over estimate the annular flow transition velocity since  $U_{SG} < U_G$ . This is especially true for high liquid rates where the annular film thickness becomes relatively large.

Chen and Spedding (1983) claim that the Taitel et al analysis is inappropriate and recommend the use of the Porter and Wong (1969) large drop terminal velocity relation to predict the annular flow transition velocity. Porter and Wong (1969) gave the large drop terminal velocity as

$$U_t = 0.317(\rho_L / \rho_g)^{1/2} \quad (4)$$

Chen and Spedding modified this empirical relation and proposed that the annular flow transition velocity is given by

$$U_{SG} = 0.317 \left[ \frac{\rho_L - \rho_g}{\rho_g} \right]^{1/2}, m/s \quad (5)$$

Chen and Spedding claim that this relation for the annular flow transition is in better agreement with the experimental data of

Taitel et al (1980) (see Figure 10), Hewitt and Roberts (1969) and Spedding and Nguyen (1980). In addition, the Chen-Spedding annular flow boundary is in better agreement with the data of other investigators shown in Taitel et al's Figure 8 and Schmidt's (1977) data.

Figure 1 shows that the Taitel et al and Chen-Spedding relations give the same behavior as a function of system pressure. It is noted that the primary difference between the two relations, other than the coefficient, is the appearance of the gas-liquid interfacial tension,  $\sigma$ , in the Taitel et al relation.

An experimental evaluation of the annular flow transition velocity as a function of  $\sigma$  is required to determine which relation is preferred since the coefficient in the Taitel et al relation can be modified to enable it to more accurately fit existing experimental data. Thus it is inappropriate to accept the annular flow transition boundary proposal of Chen and Spedding until additional experimental data is obtained to determine if there is an effect of interfacial tension.

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## Reply:

In response to the letter of Sylvester who showed that the original proposal of Taitel et al. (1980) and the modification of Chen & Spedding (1983) given here respectively as equations [1] and [2] gave the same behaviour as a function of system pressure and that the primary difference was the absence of the interfacial tension term  $\sigma$  in equation [2]. While the absolute values given by the two different relation were substantially dissimilar particularly at realistic values of surface tension, Sylvester (1984) demonstrated the usefulness of the model by modifying the coefficients of equation [1]. However, such modification require unrealistically large values of the drag coefficient.

$$U_{SG} = 3.1 \left[ \frac{\sigma g(\rho_L - \rho_g)}{\rho_g^2} \right]^{1/4} \quad (1)$$

$$U_{SG} = 0.317 \left[ \frac{\rho_L - \rho_g}{\rho_g} \right]^{1/2} m/s \quad (2)$$

where  $U_{SG}$  is the superficial gas velocity,  $\sigma$  the surface tension,  $g$  the acceleration due to gravity,  $\rho_L$  and  $\rho_g$  the liquid and gas density respectively. The following analysis is an attempt to study the interfacial tension effect.

There are a number of correlations of relevance to other somewhat related processes in which some contain the interfacial tension term  $\sigma$  while others do not and yet they represent experimental data almost just as well. Examples of some of these include flooding in counter-current flow, and the prediction of the critical heat flux in pool boiling.

In the flooding of counter-current flow, the most commonly used correlation is the Wallis correlation (Wallis (1969), Taitel et al. (1982)).

$$\left[ \frac{\rho_g U_G^2}{gD(\rho_L - \rho_g)} \right]^{1/4} + m \left[ \frac{\rho_L U_L^2}{gD(\rho_L - \rho_g)} \right]^{1/4} = C_w \quad (3)$$

where  $m \simeq 1.0$  and  $C_w \simeq 0.7 - 1.0$ , both

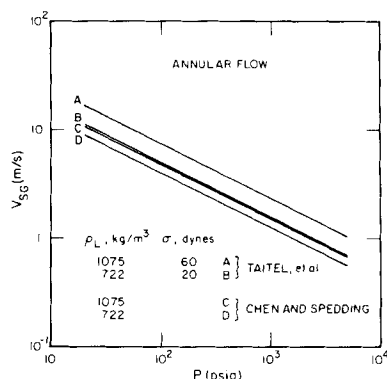


Figure 1. Effect of pressure and fluid properties in the annular flow transition velocity.

constants determined empirically from experimental data. The characteristic length  $D$  is commonly identified to the hydraulic diameter.

Another flooding correlation that has met increasing popularity is (Tien (1977), Tien & Chung (1979), Tien et al. (1980)),

$$\left[ \frac{\rho_G^{1/2} U_G}{g[\sigma(\rho_L - \rho_G)]^{1/4}} \right]^{1/2} + \left[ \frac{\rho_L^{1/2} U_L}{g[\sigma(\rho_L - \rho_G)]^{1/4}} \right]^{1/2} = \sqrt{3.2} \quad (4)$$

In the prediction of the critical heat flux (CHF),  $q_{crit}$ , in pool boiling the equation proposed by Rohsenow & Griffith (1956) is

$$\frac{q_{crit}}{\rho_G h_{fg}} = 0.0121 \left( \frac{\rho_L - \rho_G}{\rho_G} \right)^{0.6} \text{ m/s} \quad (5)$$

Note that in equation (5)  $\rho_G$  is the vapour density and  $h_{fg}$  is the latent heat of vaporization. The equation proposed by Addoms (1948) is of a somewhat similar form involving, in addition, other dimensionless groups but has 0.5 as the exponent for the parameter  $[(\rho_L - \rho_G)/\rho_G]$  instead of 0.6.

Another popular equation for the CHF is the Zuber-Kutateladze (1958-1952) equation

$$\frac{q_{crit}}{\rho_G h_{fg}} = 0.13 \frac{\sigma_g(\rho_L - \rho_G)^{1/4}}{\rho_G^2} \quad (6)$$

It is observed that equations (3) and (5) contain no  $\sigma$  term while  $\sigma$  is incorporated in equations (4) and (6), yet, equations (3) and (4) were found to represent flooding data rather well and so do equations (5) and (6) for the CHF in pool boiling. It is to be noted that equation (3) contain a characteristic length term  $D$  raised to the power of  $1/4$  and hence its effect is relatively insensitive for the range of  $D$  normally encountered.

The analysis to be carried out is an attempt to explain the apparent inconsistency, at least for the single-component, two-phase system, i.e. where the two phases are chemically the same, e.g. steam and water. The workings will be carried out for the annular flow transition correlation of Taitel et al. (1980) and the modification by Chen & Spedding (1983) given here as equation (1) and (2) respectively. It should be pointed out that it may also be repeated for the flooding correlations given as equations (3) and (4) and the CHF correlations given as equations (5) and (6).

Consider the definitive equation for the parachor (Perry (1963), Sugden (1930)),

$$\sigma = [(P/M)(\rho_L - \rho_G)]^4 \quad (7)$$

where  $\sigma$  is the surface tension in dynes/cm,  $\rho$  the density in gm/cm<sup>3</sup> and  $\rho_G$  refers to the density of the vapour and must be evaluated under saturation conditions.  $M$  is the molecular weight and the value of  $P$ , the parachor, for various substances may be found in Sugden (1930). Equation (7) gives a reasonable prediction of surface tension with an error of about 5% provided the temperature is at least 30°C lower than the critical point (Perry (1963)). Substitution of equation (7) into (1) gives

$$U_{SG} = 3.1[(P/M)(\rho_L - \rho_G)] \times \left[ \frac{g(\rho_L - \rho_G)}{\rho_G^2} \right]^{1/4} \quad (8)$$

The interfacial tension term  $\sigma$  has been eliminated and if equation (8) bears a constant ratio to equation (2), it would mean that equation (2) had in fact accounted for the interfacial tension. Assuming that equations (2) and (8) are equivalent and bearing in mind that equation (2) has the dimensions of m/s, equating  $U_{SG}$  and rearranging yields

$$(P/M)(\rho_L - \rho_G)^{3/4} = 1.83 \quad (9)$$

where  $\rho$  must be in gm/cm<sup>3</sup>.

The LHS of equation (9) is evaluated for the substances water, ammonia and carbon dioxide using data given in Haywood (1972). For water, in the saturation temperature range of 45.8 to 295.0°C, the value for the LHS of equation (9) is  $\pm 15\%$  about a mean value of 2.50. For ammonia in the range -40 to 50°C, it is  $\pm 9\%$  about a mean of 2.47 and for carbon dioxide in the range -40 to 0°C, it is  $\pm 10\%$  about a mean of 1.7. On the other hand, the numerical value of the RHS of equation (9) is equal to 1.83. Although the LHS and RHS of equation (9) are not equal, the LHS nevertheless maintains a reasonably constant value over a wide range of conditions. This in fact is in agreement with the comment of Sylvester (1984) that the equations (1) and (2) appeared to differ only in the value of the coefficients. It is not known why the value of the LHS of equation (9) calculated for carbon dioxide is closer to the value of 1.83 but differs from the calculated values for water and ammonia.

The foregoing analysis appears to have indicated that the dependence of surface

tension in equation (2), and also in equations (3) and (5) for the flooding correlation and CHF prediction respectively, although not explicitly expressed in the form of an interfacial tension term,  $\sigma$  has in fact been incorporated inherently through the factor  $(\rho_L - \rho_G)$  through the Sugden equation as shown in equation (7). It should be noted that equation (2) was in fact derived by Porter & Wong (1969) using data including those for iso-octane ( $\sigma = 21.7$  dynes/cm), hexa-decane ( $\sigma = 27.6$  dynes/cm) and water ( $\sigma = 72$  dynes/cm) and hence had the surface tension effect incorporated, though somewhat unawaringly and unintentionally.

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